Legislature which has prevented publication of the volume on coal. That interest, owing to the sudden expansion of iron manufacture, is now paramount, and the state is losing enormously by this failure to publish the material accumulated by Dr. White in extended reconnaissances during the last ten years. To those engaged in investigating the serious problems presented by the Carboniferous, the inaccessibility of this material is a misfortune.

The map shows the oil fields and productive areas of the several coal series. The limits of the Pittsburg, as determined by borings, differ from the Rogers lines as much for West Virginia as for Ohio. The geographical conditions during the formation of that bed were evidently very unlike those suggested by the older geologists.

The abundance of typographical errors is evidence that the author had no opportunity to correct the proofs, and reminds the writer of his own experience with the West Virginia State Printer almost thirty years ago, when Mr. F. B. Meek and he were made chargeable with statements which afforded some annoyance to them and much amusement to their acquaintances. JOHN J. STEVENSON.

*Introduction à la géométrie différentielle* suivant la méthode de H. GRASSMAN. Par C. Burali-Forti, professeur à l'Académie militaire de Turin. Paris, Gauthier-Villars. 1897. 8vo. Pp. xi + 165.

This volume contains a brief exposition of the geometrical calculus and some of its applications to elementary differential geometry.

The analytical geometry of Descartes (1637), operates on numbers which have an indirect relation with the geometrical elements which they represent. Leibnitz* in 1679 recognized the advantages of a geometrical calculus operating directly on geometrical elements, but the operation suggested by Leibnitz does not possess the ordinary properties of algebraic operations. The idea, however, was fruitful, and in 1797 Caspar Wessel† gave an analytical representation of direction which contains Argand's (1806) geometrical interpretation of complex numbers and several of the operations introduced by Hamilton (1843–1854) in his method of quaternions. Later the barycentric calculus (1827–1842) of Möbius and the method of equipollences (1832–1854) of Bellavitis brought forward two independent methods of geometric calculus which their authors applied to various questions of geometry and mechanics. In 1843 Hamilton published his first essay on quaternions; the complete development of this theory in 1854 gives a complete geometrical calculus which finds at present its most extensive applications in mathematical physics. The works of Hamilton were preceded by the Ausdehnungslehre (1844), of H. Grassmann which, in the power and simplicity of its operations, surpasses all other known forms of geometrical calculus. The method of exposition adopted by Grassmann is exceedingly abstract and this fact has stood stubbornly in the way of the general adoption of the Ausdehnungslehre to such an extent that we use to-day the bar tric calculus, the theory of equipollences, quaternions, or the Cartesian geometry, for the resolution of geometric questions which are capable of much more simple resolution by the methods of Grassmann. These classic objections to Grassmann's exposition have been met recently by Peano* who has given concrete geometric interpretations to the forms and operations of the Ausdehnungslehre. There is a splendid account of the importance of this discipline in geometry, mechanics and physics to be found in the historical memoir of Schlegel.†

M. Burali-Forti gives the elements of Grassmann's calculus as reconstructed by Peano. The latter took the idea of a tetrahedron as his starting point and defined the product of two and three points; he then defined the products of these elements by numbers and finally gave definitions of the sums of these products. The theory of forms of the first order gives the barycentric calculus and that of vectors; the geometric forms of the second order represent straight lines, orientations, and systems of forces

* Leibnitz, *Math. Schriften*, II., V., Berlin, 1849.

† Caspar Wessel, Om Directionens analytiske Betegning, March 10, 1797; published by the Denmark Academy of Sciences, Copenhagen, 1897.

* Peano, Calcolo geometrico, Turin, 1888.

† Schlegel, Die Grassmann'sche Ausdehnungslehre, *Zeitschrift für Math. und Physik*, 1896.

applied to rigid bodies; the forms of the third order represent planes and the plane at infinity. Among the operations, the progressive and regressive products give the geometric operations of projecting and cutting; the inner product gives the orthogonal projections and the elements which we designate in mechanics by the terms moment, work, *et cetera*.

In ordinary differential geometry simple properties most frequently yield themselves only after very complicated calculations. This complication is due in general to the use of coördinates; with these coördinates algebraic transformations are made on numbers in order to obtain certain formulæ, namely, invariants, which are susceptible of geometric interpretations. On the other hand the geometrical calculus makes no use whatever of coördinates; it operates directly on the geometric elements; each formula which it produces is an invariant, capable of a simple geometric interpretation and leading directly to the graphic representation of the elements considered. Burali-Forti's work, though by no means a pioneer in the application of Grassmann's theories to differential geometry (note for example the memoirs of the younger Grassmann in the theory of curves and surfaces), shows the elegant power and simplicity of the geometrical calculus in elementary differential geometry and points the student to a vast field of transformations and researches in higher geometry.

The work is designed after the following plan which exhibits the skeleton of its contents:

I. The geometric forms.—1° Definitions and rules of calculus:—tetrahedron, geometric forms, equality of forms, points, segments, triangles, sum and product by a number, progressive product; 2° Vectors and their products:—vectors, bivectors, trivectors, rotation, operation index; 3° Reduction of forms:—forms of the first order, forms of the second order, forms of the third order, projective elements, identity between forms of the first order; 4° Regressive products:—forms of the second and third orders, forms of the third order, general properties of products, duality, regressive products in a projective plane; 5° Coördinates.

II. Variable forms.—1° Derivatives:—definitions, limit of a form, limit of a projective element, derivatives, mean forms, Taylor's formula, continuous forms; 2° Lines and envelopes:—lines and envelopes of straight lines on a projective plane, space curves and envelopes of planes; 3° Ruled surfaces:—ruled surfaces in general, skew ruled surfaces, developable surfaces; 4° Frenet's formulæ:—arcs, curvature and radius of curvature, torsion and radius of torsion, formulæ of Frenet, spherical indicatrix and angle of contingence.

III. Application.—1° Helix; 2° Surfaces ruled relative to a curve—polar surface, rectifying surface, surface of principal normals, surface of binormals, skew ruled surfaces whose line of striction is given, developable ruled surface described by a straight line whose position is fixed with regard to the tetrahedron PTNB; 3° Orthogonal trajectories:—orthogonal trajectories of the generatrices of a ruled surface, evolutes, involutes, orthogonal trajectories of planes of an envelope; 4° Curves of Bertrand.

NOTES.—1° Forms which are functions of two or more variables; 2° Tangent plane; 3° Differential parameter of first order; 4° Curvilinear coördinates.

E. O. LOVETT.

PRINCETON, NEW JERSEY.

*Chemical Experiments*. By JOHN F. WOODHULL, Professor of Physical Science, Teachers College, Columbia University, and M. B. VAN ARSDALE, Instructor in Physical Science in Horace Mann School and Assistant in Teachers College. New York, Henry Holt & Co. 1899. Pp. 136. Price, 50 cents.

This book gives a series of very elementary experiments dealing chiefly with the elements oxygen, hydrogen, chlorine, sulphur, nitrogen and carbon. The apparatus recommended for the experiments is simple, and in several cases, quite ingenious. For pupils of a certain grade the book will doubtless prove useful, but the introduction of a few more quantitative experiments designed to illustrate fundamental principles seems desirable.

*A Laboratory Outline of General Chemistry*. By ALEXANDER SMITH. Chicago, Kent Chemical Laboratory of the University of Chicago. 1899. Pp. xii+90.

The work before us represents a very dis-